COBEM 99
XV Congresso Brasileiro de Engenharia Mecânica
15th Brazilian Congress of Mechanical Engineering
22 - 26 de Novembro de 1999 / November 22 - 26, 1999 Águas de Lindóia, São Paulo,

MODELING AND SIMULATION OF DELAMINATION IN COMPOSITE TUBES

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Abstract. *Delamination is a phenomenon characterized by the loss of adhesion between two adjacent laminae. This is a damage process frequently observed in composite materials and it may cause either loss of structural stiffness or total failure of the laminate. This contribution presents a model to describe composite delamination. The proposed model considers a laminate with a finite thickness interlayer. Interlaminar stresses are evaluated from a modified lamination theory. This result is used as input in the constitutive adhesion model which describes the damage evolution of the interlayer. An iterative numerical procedure is developed, solving the model equations separately. This work considers numerical simulations of a laminated tube as an application of the proposed general formulation.*

Keywords: *Composite Materials, Delamination, Constitutive equations.*

1. INTRODUCTION

The basic building block of a composite material is the lamina, which usually consists of a reinforced fiber matrix. Several laminae are usually bonded together to act as an integral structural element denoted as laminate. The degradation modes of composite laminates can be split into two classes: intralaminar damages and delamination. Intralaminar damage includes transverse matrix cracking, fiber-matrix debonding and fiber ruptures. On the other hand, delamination is a phenomenon characterized by the loss of adhesion between two adjacent laminae. This is a damage process frequently observed in composite material and it may cause either loss of structural stiffness or total failure of the laminate. Delamination may be caused by interlaminar stress concentration, which occurs either in the neighborhood of the free edge or around loaded holes of the composite (Point & Sacco, 1996).

The study of delamination process may be carried out by two different approaches. The first is the fracture mechanics, which considers the failure modes of the material. The second approach considers phenomenological constitutive equations to describe the interlaminar behavior. Usually, the interlayer is considered as a surface, neglecting its thickness. Therefore, delamination is characterized by the loss of contact between the laminae (Point & Sacco, 1996; Fremond *et al.*, 1996; Tien, 1990; Point, 1989; Fremond, 1985, 1987, 1988).

This contribution presents a model to describe composite delamination. The proposed model considers a laminate with a finite thickness interlayer. Interlaminar stresses are evaluated from a modified lamination theory proposed by Bai *et al*. (1997). This result is used as input in a constitutive adhesion model which describes the damage evolution of the interlayer. This constitutive model is based on the adhesion model proposed by Fremond and co-workers (Point & Sacco, 1996; Fremond *et al.*, 1996; Tien, 1990; Fremond, 1985). An iterative numerical procedure is developed, solving the model equations separately. Finally, this work considers numerical simulations of a laminated tube as an application of the proposed general formulation.

2. MODIFICATION OF CLASSICAL LAMINATION THEORY

The determination of interlaminar stresses is very important to analyze delamination problem. This contribution evaluates the interlaminar stresses by considering a modification of classical lamination theory proposed by Bai *et al*. (1997). With this aim, consider a two-layer laminated element, each with thickness h and an interlayer with finite thickness δ, as depicted in Fig.1.

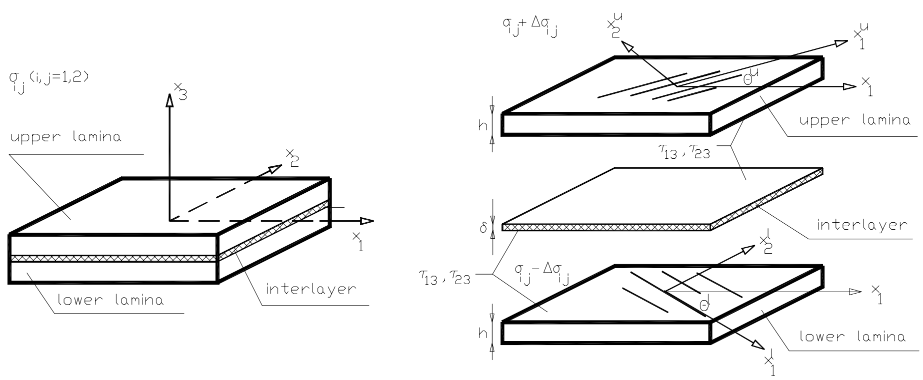

Figure 1 - Laminate with interlayer.

Interlaminar deformations, ε_{13} and ε_{23}, are resulted from interlaminar stress, τ_{13} and τ_{23}, induced by the stiffness mismatch between laminae. Since the interlaminar deformation in each lamina is much smaller than at each interface due to imperfect interfacial bonding, it is assumed that there is no interlaminar deformation in each lamina and all interlaminar deformation occurs in the interlayer. Hence, consider the two-layer laminate with rectangular coordinates (x_1, x_2, x_3) where x_1-x_2 plane coincides with the mid-plane of the laminate. The in-plane stresses on each lamina consist of the sum of the stress σ_{ij} (i, j = 1,2), due to external loads, and the constraint stress $\Delta\sigma_{ij}$ (i, j = 1,2) provided by its adjacent laminae.

$$\sigma_{ij}^{u} = \sigma_{ij} + \Delta\sigma_{ij} \quad \text{and} \quad \sigma_{ij}^{l} = \sigma_{ij} - \Delta\sigma_{ij} \tag{1}$$

where σ_{ij}^{u} and σ_{ij}^{l} are the stresses on the upper and lower laminae, respectively.

By considering the compliance tensor of the laminae, S_{ijkl}^{u} and S_{ijkl}^{l}, it is possible to write the stress-strain relation on each lamina,

$$\varepsilon_{ij}^{u} = S_{ijkl}^{u}\,\sigma_{kl}^{u} \quad \text{and} \quad \varepsilon_{ij}^{l} = S_{ijkl}^{l}\,\sigma_{kl}^{l} \tag{2}$$

Now, taking into account the stress transformation tensor, T_{ijkl}^{u} and T_{ijkl}^{l}, one obtains the strain on each lamina (Bai *et al.*, 1997),

$$\varepsilon_{mn}^{u} = T_{mnkl}^{u} S_{klpq}^{u} T_{pqrs}^{u} (\sigma_{rs} + \Delta\sigma_{rs}) \quad \text{and} \quad \varepsilon_{mn}^{l} = T_{mnkl}^{l} S_{klpq}^{l} T_{pqrs}^{l} (\sigma_{rs} - \Delta\sigma_{rs}) \tag{3}$$

At this point, it is possible to evaluate the interlaminar strains as follows,

$$2\varepsilon_{mn}^{u/l} = (\varepsilon_{mn}^{u} - \varepsilon_{mn}^{l}) = B_{mnrs}\Delta\sigma_{rs} + A_{mnrs}\sigma_{rs} \tag{4}$$

where

$$B_{mnrs} = T_{mnkl}^{u} S_{klpq}^{u} T_{pqrs}^{u} + T_{mnkl}^{l} S_{klpq}^{l} T_{pqrs}^{l} \quad \text{and} \quad A_{mnrs} = T_{mnkl}^{u} S_{klpq}^{u} T_{pqrs}^{u} - T_{mnkl}^{l} S_{klpq}^{l} T_{pqrs}^{l} \tag{5}$$

According to the linear shear slip theory (Lu & Liu, 1992), the difference between the displacements of the upper and the lower surfaces of the interlayer is given by,

$$\Delta u(x_1, x_2, x_3) = \frac{\delta}{G}\tau_{13}(x_1, x_2) \quad \text{and} \quad \Delta v(x_1, x_2, x_3) = \frac{\delta}{G}\tau_{23}(x_1, x_2) \tag{6}$$

where Δu and Δv are the displacement difference in the x_1 and x_2 directions, respectively. The constant G is the shear modulus of the interlayer.

For continuity conditions, the in-plane displacements of the upper and the lower laminae must be equal to the displacements of the upper and lower surfaces of the interlayer. Therefore, assuming infinitesimal strain hypothesis and linear elastic relations, it is possible to write,

$$\begin{aligned} \varepsilon_{11}^{u/l} &= \frac{\delta}{2G}\,\frac{\partial\tau_{13}}{\partial x_1} \\ \varepsilon_{22}^{u/l} &= \frac{\delta}{2G}\,\frac{\partial\tau_{23}}{\partial x_2} \\ \varepsilon_{12}^{u/l} &= \frac{\delta}{4G}\left(\frac{\partial\tau_{23}}{\partial x_1} + \frac{\partial\tau_{13}}{\partial x_2}\right) \end{aligned} \tag{7}$$

Establishing the equilibrium on the lamina element, the following equations are obtained

$$\begin{aligned} \left(\frac{\partial\Delta\sigma_{11}}{\partial x_1} + \frac{\partial\Delta\tau_{12}}{\partial x_2}\right)h - \tau_{13} &= 0 \\ \left(\frac{\partial\Delta\sigma_{22}}{\partial x_2} + \frac{\partial\Delta\tau_{12}}{\partial x_1}\right)h - \tau_{23} &= 0 \end{aligned} \tag{8}$$

Using these results in the equations of interlaminar strain, Eq. (7), and then in Eq. (4), one obtains the constraint stress $\Delta\sigma_{11}$, $\Delta\sigma_{22}$ and $\Delta\tau_{12}$ from the following set of differential equations:

$$(B_{1111}\Delta\sigma_{11} + B_{1122}\Delta\sigma_{22} + B_{1112}\Delta\tau_{12}) - \frac{\delta}{G}\left(\frac{\partial^2\Delta\sigma_{11}}{\partial x_1^2} + \frac{\partial^2\Delta\tau_{12}}{\partial x_1 \partial x_2}\right)h = -(A_{1111}\sigma_{11} + A_{1122}\sigma_{22} + A_{1112}\tau_{12}) \quad (9)$$

$$(B_{2211}\Delta\sigma_{11} + B_{2222}\Delta\sigma_{22} + B_{2212}\Delta\tau_{12}) - \frac{\delta}{G}\left(\frac{\partial^2\Delta\sigma_{22}}{\partial x_2^2} + \frac{\partial^2\Delta\tau_{12}}{\partial x_1 \partial x_2}\right)h = -(A_{2211}\sigma_{11} + A_{2222}\sigma_{22} + A_{2212}\tau_{12}) \quad (10)$$

$$(B_{1211}\Delta\sigma_{11} + B_{1222}\Delta\sigma_{22} + B_{1212}\Delta\tau_{12}) - \frac{\delta}{2G}\left(\frac{\partial^2\Delta\sigma_{11}}{\partial x_1 \partial x_2} + \frac{\partial^2\Delta\sigma_{22}}{\partial x_1 \partial x_2} + \frac{\partial^2\Delta\tau_{12}}{\partial x_1^2} + \frac{\partial^2\Delta\tau_{12}}{\partial x_2^2}\right)h =$$
$$= -(A_{1211}\sigma_{11} + A_{1222}\sigma_{22} + A_{1212}\tau_{12}) \quad (11)$$

3. ADHESION MODEL

The thermodynamic state of a solid is completely defined by the knowledge of the state variables. Constitutive equations may be formulated within the formalism of continuum mechanics and thermodynamics of irreversible processes, by considering thermodynamic forces, defined from the Helmholtz free energy, ψ, and thermodynamic fluxes, defined from the pseudo-potential of dissipation, ϕ (Lemaitre & Chaboche, 1990). The adhesion model here proposed is based on the constitutive model proposed by Fremond and co-workers (Point & Sacco, 1996; Fremond *et al.*, 1996; Tien, 1990; Point, 1989; Fremond, 1985).

With this aim, consider a variable associated with the relative displacement between two points of the interlayer, with the same coordinate (x_1, x_2), r. In this article, the following definition is considered,

$$r = \sqrt{\Delta u^2 + \Delta v^2} \quad (12)$$

In order to evaluate adhesion, a damage variable γ is introduced. This variable is associated with bonded surfaces and assumes the following values: $\gamma = 0$, when there is total adhesion; $0 < \gamma < 1$, when the adhesion is partial; $\gamma = 1$, when there is no adhesion. As a matter of fact, this variable represents two kinds of damage associated with the adhesive damage between upper and lower laminae and the interlayer. The damage variables associated with intralaminar behavior are not included in this model.

Therefore, the interlayer state is defined by the pair (r, γ), which represents the state variables of the delamination phenomenon. At this point, consider a Helmholtz free energy with the form,

$$\psi(r,\gamma) = \frac{k}{2}(1-\gamma)^2 r^2 + \frac{a}{2} r^2 + I_C(\gamma) \quad (13)$$

where k and a are constants. I_C represents the indicator function associated with the set C defined as follows

$$C = \{ \gamma : \; 0 \leq \gamma \leq 1, \; \dot{\gamma} \geq 0 \} \tag{14}$$

The thermodynamic forces are given by (Lemaitre & Chaboche, 1990),

$$\begin{aligned} X^R &= \partial_r \psi(r,\gamma) = \left[k(1-\gamma)^2 + a\right] r \\ Y &= -\partial_\gamma \psi(r,\gamma) = k(1-\gamma) r^2 + \partial_\gamma I_C \end{aligned} \tag{15}$$

where $\partial_i \psi$ is the sub-differential of the Helmholtz free energy with respect to variable i (Rockafellar, 1970). Now, consider the dual of the potential of dissipation,

$$\phi^*\left(X^R, Y\right) = \frac{b}{2} Y^2 + \frac{c}{2}\left(X^R\right)^2 + I_W(Y) \tag{16}$$

where b and c are constants. I_W represents the indicator function associated with the set W defined as follows

$$W = \{ Y : \; Y \geq 0 \} \tag{17}$$

The evolution equations of the state variables are given by the following definitions (Lemaitre & Chaboche, 1990),

$$\begin{aligned} \dot{r} &= \partial_{X^R} \phi^*\left(X^R, Y\right) = c X^R \\ \dot{\gamma} &= \partial_Y \phi^*\left(X^R, Y\right) = bY + \partial_Y I_W \end{aligned} \tag{18}$$

where $\partial_i \phi^*$ is the sub-differential of the dual of potential of dissipation with respect to a thermodynamic force, i. Since the pseudo-potential of dissipation, or its dual, is convex, positive and vanishes at the origin, the Clausius-Duhen inequality (Eringen, 1967),

$$\sigma : \dot{\varepsilon} - \rho(\dot{\psi} + s\dot{T}) \geq 0 \, , \tag{19}$$

is automatically satisfied if the entropy is defined as $s = -\partial \psi / \partial T$.

4. NUMERICAL PROCEDURE

The numerical procedure here proposed has two parts. On the first, interlaminar stresses are evaluated using the modification of the classical lamination theory proposed by Bai *et al.* (1997). The next step of solution consists on evaluating the evolution of the state variables of the adhesion model. An iterative procedure assures the convergence of the procedure.

The determination of interlaminar stresses may be either analytical or numerical, solving Eq. (9-11). From this solution, it is possible to calculate relative displacements which are used as an input on the adhesion model. Time discretization is necessary to evaluate the evolution of state variables. By considering the implicit Euler algorithm, the following equations are written,

$$\left(X^{R}\right)_{n}^{i} = \left[k \left(1-\gamma_{n}^{i}\right)^{2} + a \right] r_{n}^{i}$$

$$Y_{n}^{i} = k\left(1-\gamma_{n}^{i}\right)\left(r_{n}^{i}\right)^{2} + \partial_{\gamma} I_{C}$$

$$\gamma_{n}^{i} = \gamma_{n-1}^{i} + \Delta t\, [b\ Y_{n}^{i} + \partial_{Y} I_{W}] \tag{20}$$

where the superscript i is associated with space, while subscript n is associated with time instant. The sub-differential $\partial_{\gamma} I_{C}$ is numerically treated by considering the projection of the variable γ on the set C, while $\partial_{Y} I_{W}$ considers the projection of Y on the set W.

An iterative numerical procedure is employed until some convergence criterion is satisfied. In this article, one considers that the pair (r, γ), at a given point and in two subsequent time instants, are very close.

In order to analyze post-delamination behavior a generic point j, which is the outer non-delaminated point, is considered (Fig.2). The relative displacement of the points between point j and the free edge must be evaluated by an alternative procedure. One conceives that the relative displacement r is calculated by spatial numerical integration of the strain. With this assumption, the displacements after delamination are calculated as follows,

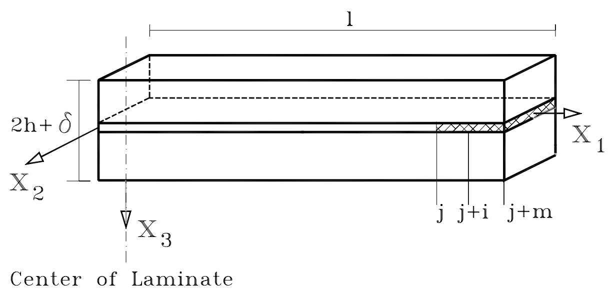

Figure 2 - Laminate showing a delaminated region.

$$u^{u} = \int_{x_j}^{x_{j+m}} \varepsilon_{11}^{u}\, dx \quad \text{and} \quad v^{u} = 2 \int_{x_j}^{x_{j+m}} \varepsilon_{12}^{u}\, dx \tag{21}$$

$$u^{l} = \int_{x_j}^{x_{j+m}} \varepsilon_{11}^{l}\, dx \quad \text{and} \quad v^{l} = 2 \int_{x_j}^{x_{j+m}} \varepsilon_{12}^{l}\, dx \tag{22}$$

which permits to obtain the relative displacements,

$$\Delta u = u^{u} - u^{l} \quad \text{and} \quad \Delta v = v^{u} - v^{l} \tag{23}$$

Hence, the relative displacement of a generic point on the delaminated region, is given by

$$r_{j+i} = r_{j} + \left(\sqrt{\Delta u^{2} + \Delta v^{2}}\right)_{j}^{j+i} \tag{24}$$

This is a simplified procedure, which represents a first approach of the problem. Nevertheless, it should be pointed out that a more detailed analysis of this problem, out of the scope of this contribution, must be carried out to validate it.

5. LAMINATED TUBE

As an application of the proposed model, an anti-symmetric two-layer angle ply laminated tube, depicted in Fig.3, is considered. The analysis is restrict to situations where lamina response occurs on elastic domain, and that the composite failure occurs by delamination. With this assumption, either lamina or interface rupture cannot occur. This hypothesis is confirmed using *von Mises* criterion for the interlayer and *Tsai-Hill* criterion for the laminae (Gibson, 1994).

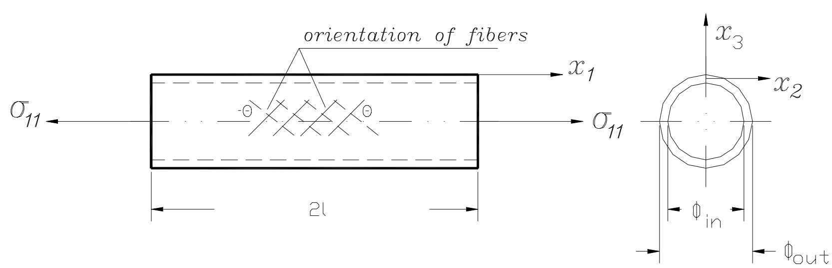

Figure 3 - Anti-symmetric laminated tube [+θ,-θ].

By considering the same assumption employed by Bai *et al.* (1997), that is, each lamina has the same geometrical and material properties, Eq. (9-11) are simplified resulting on the following relations for the interlaminar stresses

$$\tau_{13} = \frac{1}{2} C_1 D_1 \left(e^{D_1 x_1} - e^{-D_1 x_1} \right) h$$
$$\tau_{23} = \frac{1}{2} C_2 D_2 \left(e^{D_2 x_1} - e^{-D_2 x_1} \right) h \tag{25}$$

where

$$C_1 = \frac{\left[A_{1112} - (B_{1122}\, A_{2212} / B_{2222}) \right]}{\pi\, h\, D_1^2 \left(e^{-D_1 l} + e^{D_1 l} \right)} \tau_{12}$$

$$C_2 = \frac{2 \left(A_{1211}\, \sigma_{11} + A_{1222}\, \sigma_{22} \right)}{\pi\, h\, D_2^2 \left(e^{-D_2 l} + e^{D_2 l} \right)}$$

$$D_1 = \sqrt{ \frac{1}{\eta\, h} \left(B_{1111} - \frac{B_{1122}\, B_{2211}}{B_{2222}} \right) }$$

$$D_2 = \sqrt{\frac{2B_{1212}}{\eta h}} \tag{26}$$

As an example, one considers a [+30°/-30°] 60mm long laminated tube with 15mm of internal diameter and 2mm of thickness. The material is AS/3501 whose properties are presented in Table 1. The constitutive properties of the interlayer are presented in Table 2. One has also considered a cyclic tensile stress load, depicted in Fig.4.

Table 1. Material properties.

Material	E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	υ_{12}
AS/3501	138,0	9,0	6,9	0,30

Table 2. Interface constitutive properties.

k [10^9 N.m^{-3}]	a [10^9 N.m^{-3}]	b [(m^2)(J.s^2)$^{-1}$]	c [m^3 (N.s^2)$^{-1}$]	δ/G [m.(Pa)$^{-1}$]
500	7	100	5.10^{-12}	10^{-13}

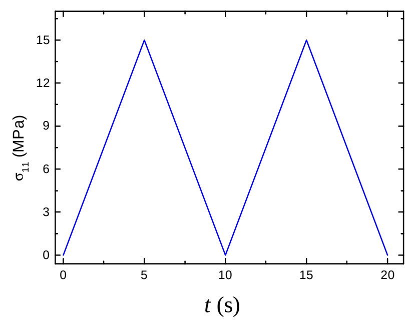

Figure 4 - Cyclic tensile stress load.

An analysis of state variables and thermodynamic forces is now in order. Time evolution of the relative displacement r, is presented in Fig.5a. After delamination process is completed, the laminae are debonded, and there is a great relative displacement increase. This is a consequence of contact loss between the laminae that causes a loss of resistance. The thermodynamic force X^R is associated with the relative displacement and represents the contact stress between lamina and interlayer (Fremond *et al.*, 1996). As it can be seen in Fig.5b, this variable has a maximum value before delamination begins to occur and, after this, decreases. It means a loss of resistive stress. When the laminae are debonded, there are small values of X^R meaning that some contact stress remains to be provided by the adhesive (Tien, 1990; Fremond *et al.*, 1996).

Damage variable evolution is presented in Fig.5c and permits to visualize the delamination evolution. When $\gamma = 1$, delamination is completed and the laminae debonds. It is clear that delamination process starts at free edge and propagates to interior points. This result is in close agreement with experimental analysis (Pipes and Pagano, 1970) and is a consequence of asymptotic growth of interlaminar shear stresses on free edge region (Wang & Choi, 1982). Thermodynamic force Y is associated with damage variable γ and represents the energy necessary to promote the delamination process. As Fig.5d shows, the maximum value of this variable is at the free edge meaning the high energy associated with delamination of this point. After delamination of the free edge, the Y value tends to decrease in other points. Observing time evolution of Y at some particular point, this variable presents a maximum value and then becomes to decrease as delamination takes place. When the laminae are debonded, Y assumes null values.

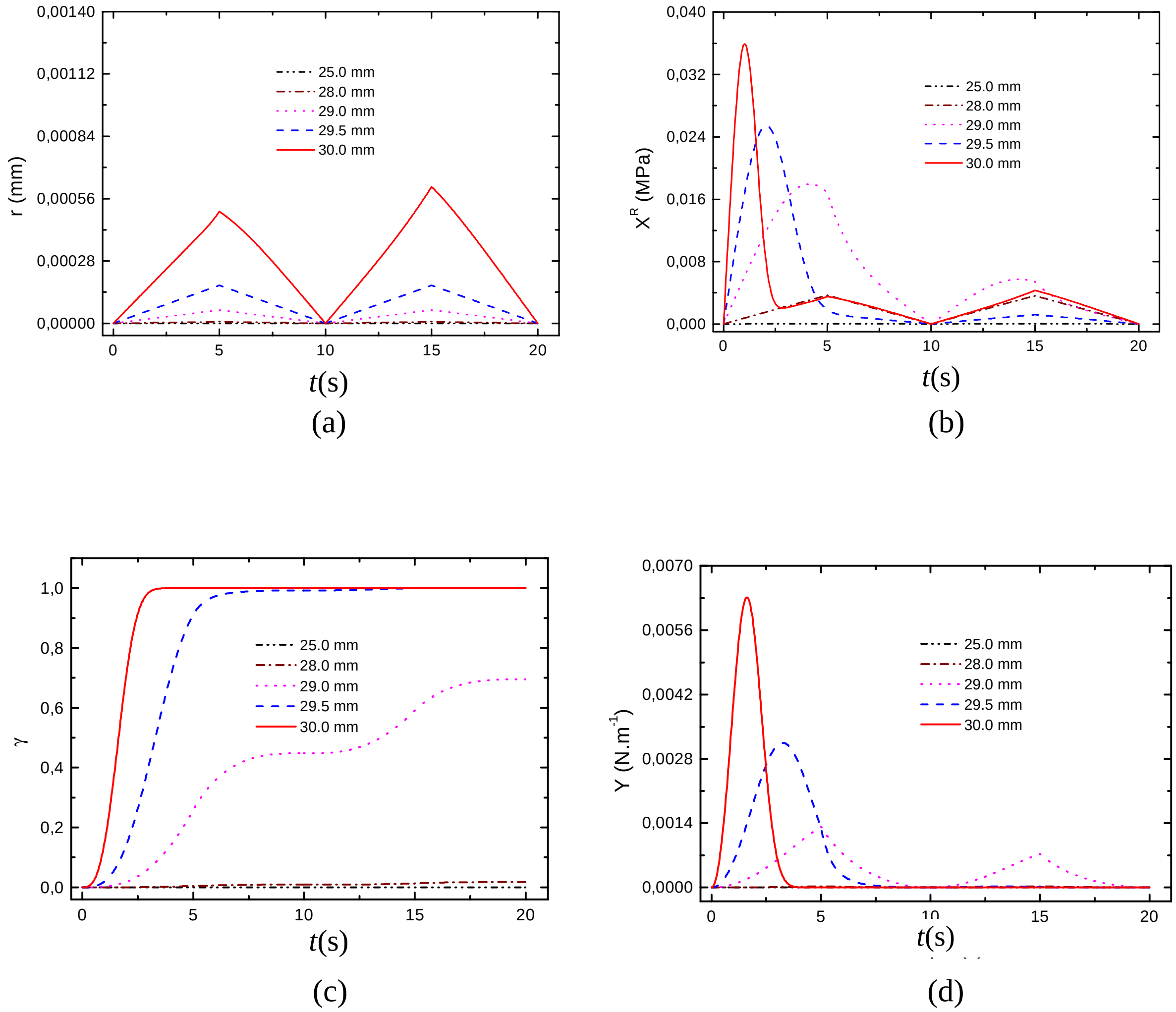

Figure 5 - Time evolution of state variables and thermodynamic forces for [+30°/-30°] AS/3501 laminated tube, subjected to cyclic tensile stress. a) r ; b) X^R; c) γ; d) Y.

6. CONCLUSIONS

This contribution reports on a model to describe delamination on laminated composite materials. The proposed model considers a laminate with a finite thickness interlayer. Interlaminar stresses are evaluated from a modified lamination theory. This result is used as an input in a constitutive adhesion model which describes the damage evolution of the interlayer. An iterative numerical procedure is developed, solving the model equations separately. Numerical simulations of a laminated tube are considered as an application of the proposed general formulation. An analysis of state variables and thermodynamic forces are presented, explaining their physical meaning. Numerical results show that the model is capable of predicting qualitatively the delamination phenomenon. Nevertheless, experimental analysis is necessary for quantitative validation of the proposed model.

Acknowledgements

The authors would like to acknowledge the support of the Brazilian Research Council (CNPq).

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